TAKE HOME EXAM 3

Solve the following problems showing all your work for full credit.

1. Find the exact value of:
   a) (5 pts.) \( \cos(\alpha - \beta) \) if \( \cos \alpha = \frac{1}{3} \), \( \alpha \) is in quadrant IV and \( \cos \beta = -\frac{1}{4} \), \( \beta \) is in quadrant II.

   \( \beta \) is in quadrant II.

   b) (5 pts.) \( \tan(\alpha + \beta) \) if \( \sin \alpha = -\frac{3}{5} \), \( \alpha \) is in quadrant III and \( \cos \beta = -\frac{5}{12} \), \( \beta \) is in quadrant II.

2. Use a sum or difference identity to find
   a) (3 pts.) \( \sin 165^\circ \) exactly.

   b) (3 pts.) \( \tan \frac{19\pi}{12} \) exactly.

3. Simplify the expression:
   a) (5 pts.) \( \frac{(\sin x - \sin y)^2 + (\cos x - \cos y)^2 - 2}{\sin(x - y)} \) =

   b) (4 pts.) \( \frac{\tan^4 x - 1}{\tan^2 x - 1} = \)
4. Verify each identity:
   a) (5 pts.) \( \cos^4 x - \sin^4 x = \cos 2x \)

   b) (5 pts.) \( \cos^2 2x = \frac{\tan 4x + \sin 4x}{2\tan 4x} \)

   c) (5 pts.) \( \tan^2 \left( \frac{x}{2} \right) = \frac{1 - \cos x}{1 + \cos x} \)

5. Use half-angle identity to evaluate
   a) (3 pts.) \( \sin \left( -22.5^\circ \right) \) exactly.

   b) (3 pts.) \( \cos \left( -\frac{3\pi}{8} \right) \) exactly.

6. Given that \( \cos \theta = \frac{2}{3} \) and the terminal side is in quadrant IV, find:
   a) (2 pts.) \( \cos \left( \frac{\pi}{2} + \theta \right) = \)

   b) (2 pts.) \( \sin 2\theta = \)

   c) (2 pts.) \( \cos 2\theta = \)

   d) (3 pts.) \( \sin \frac{\theta}{2} = \)
7. Find the exact value of the expression whenever it is defined:
   a) (2 pts.) \( \cos^{-1}\left(-\frac{1}{2}\right) \) in radians
   b) (2 pts.) \( \arcsin\left(\frac{\sqrt{3}}{2}\right) \) in degrees
   c) (3 pts.) \( \tan^{-1}(\tan \frac{3\pi}{4}) = \)
   d) (3 pts.) \( \csc(\cos^{-1}\left(\frac{1}{5}\right)) = \)
   e) (4 pts.) \( \cos(\sin^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right)) = \)

8. Solve, finding all solutions of the equation in the interval \([0, 2\pi)\):
   a) (4 pts.) \( 4\cos^2 x = 1; \)
   b) (5 pts.) \( \cos x(2\cos x + 7) = 4; \)
c) (5 pts.) \( \sqrt{3} \cos x + \cos x \cot x = 0 \).

d) (5 pts.) \( 2 \sin x \tan x - \tan x = 1 - 2 \sin x \).

9. Solve the given triangles (round to two decimal places):
   a) (4 pts.) \( \alpha = 45^\circ, \gamma = 35^\circ, c = 10 \text{ cm}; \)
   b) (4 pts.) \( a = 5 \text{ in}, b = 7 \text{ in}, c = 9 \text{ in}. \)

10. (4 pts.) A plane flew due north 450 mph for four hours. A second plane, starting at the same point and at the same time, flew southeast at an angle 135° clockwise from due north at 375 mph for four hours. At the end of the four hours, how far apart were the two planes? Round to the nearest mile.