TAKE-HOME EXAM 1

Solve the following problems showing all your work for full credit.

1. Find the domain of the functions and express it in interval notation:
   a) (2 pt.) \( f(x) = \frac{x+2}{x^2-9}; \)
   b) (2 pt.) \( \sqrt{4-x^2}; \)
   c) (2 pt.) \( f(x) = \frac{x-3}{(x-2)(x+1)}. \)

2. Which of the following functions are odd? Even? Neither even nor odd?
   a) (1 pt.) \( f(x) = \frac{5x^5}{3x^2-1}; \)
   b) (1 pt.) \( g(x) = |x^3|; \)
   c) (1 pt.) \( h(x) = 3x^4 + 5\cos x. \)

3. A function \( f \) is defined by \( f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 1, & 0 \leq x \leq 1 \\ 3-x, & 1 < x \end{cases} \)
   a) (3 pts.) Evaluate \( f(-3), f(1), \) and \( f(6); \)
   b) (3 pts.) Make a hand-drawn graph of \( f(x). \)
4. The graph of a function $y = f(x)$ is shown below. No formula for $f(x)$ is given.

a) (2 pts.) Find the domain and range of $f(x)$.

b) (3 pts.) Sketch the graph of the inverse function $f^{-1}(x)$;

c) (3 pts.) Sketch the graph of the function $g(x) = 2f(x + 1) - 1$.

5. (2 pts.) Let $f(x) = x^2 - 2x$ and $g(x) = x + 2$. Find $f \circ g$ and $g \circ f$.

6. Calculate each of the following without using a calculator.
   a) (1 pt.) $\sin 330^\circ$
   b) (1 pt.) $\cos \frac{8\pi}{3}$
   c) (1 pt.) $\tan \frac{4\pi}{3}$
7. (13 pts.) Consider the following graph of a function \( f(x) \):

Find the limit, if it exists.

a) \( \lim_{x \to -4} f(x) = \)

b) \( \lim_{x \to -3^-} f(x) = \)

c) \( \lim_{x \to -3^+} f(x) = \)

d) \( \lim_{x \to 3^-} f(x) = \)

e) \( f(-3) = \)

f) \( \lim_{x \to 3^+} f(x) = \)

g) \( \lim_{x \to 2^-} f(x) = \)

h) \( \lim_{x \to 2^+} f(x) = \)

i) \( \lim_{x \to 2} f(x) = \)

j) \( \text{Find } f(2) = \)

k) Is \( f(x) \) continuous at \(-3\)?

l) Is \( f(x) \) continuous at \(2\)?

m) Is \( f(x) \) continuous at \(6\)?

8. Find the limit if it exists:

a) (1 pt.) \( \lim_{x \to 1} \frac{-2x^2+3}{5x-1} = \)

b) (1 pt.) \( \lim_{x \to 3} \cos \frac{\pi x}{12} = \)

c) (2 pts.) \( \lim_{x \to -5} \frac{x^2-25}{x+5} = \)

d) (2 pts.) \( \lim_{x \to 3} \frac{x^2-6x+9}{x-3} = \)

e) (3 pts.) \( \lim_{x \to 4} \frac{\sqrt{x-2}}{x-4} = \)

f) (3 pts.) \( \lim_{x \to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} = \)

g) (3 pts.) \( \lim_{x \to 0} \frac{1-(\cos 3x)^2}{(\tan 5x)^2} = \)
h) (2 pts.) \( \lim_{\theta \to 0} \frac{\cos(\pi - \theta) \tan \theta}{\theta} = \)

i) (1 pt.) \( \lim_{x \to -\infty} \frac{x^2 - 3x + 9}{x - 3} = \)

j) (1 pt.) \( \lim_{x \to \infty} \frac{-2x + 5}{x^2 - 2x + 6} = \)

k) (1 pt.) \( \lim_{x \to -\infty} \frac{-x^2 - 5x + 11}{(x-4)(x+1)} = \)

l) (1 pt.) \( \lim_{x \to 1^-} \frac{x^2 - 3x + 1}{x - 1} = \)

m) (1 pt.) \( \lim_{x \to 3^+} \frac{x^2 - 6x + 7}{(x-3)(x-1)} = \)

9. Find the limit L. Then use the \( \varepsilon - \delta \) definition to prove that the limit is L.
   a) (7 pts.) \( \lim_{x \to -2} (3x + 2) = \)

   b) (7 pts.) \( \lim_{x \to 2} (x^2 - 5x) = \)

10. (4 pts.) Explain why the function \( f(x) = x^3 + 3x - 2 \) has a zero in the interval \([0, 1]\).
11. (6 pts.) Find the asymptotes (if any) of the functions:
   a) \( g(x) = \frac{3x^2}{x^2 - 4}, \)
   b) \( h(x) = \frac{x^2}{x - 2}. \)

12. (3 pts.) Find the constant \( a \) such that the function \( f(x) = \begin{cases} x^2 - 1 , & x \leq 3 \\ 4x + a , & x > 3 \end{cases} \) is continuous on the entire real line.

13. (4 pts.) Sketch the graph of a function that has domain \([0,4]\) and is continuous on \([0,2)\) and \([2,4]\) but is not continuous on \([0,4]\).

14. (2 pts.) At what points, if any, is the function \( f(x) \) undefined? Discontinuous? If the function is discontinuous at some point(s), justify your answer?
   \[
   f(x) = \begin{cases} 
   3x^2 - 12 & , \quad x \leq -2 \\
   \sqrt{x} + 2 & , \quad -2 < x \leq 1 \\
   -x + 3 & , \quad 1 < x < 2 \\
   1 & , \quad 2 < x 
   \end{cases}
   \]

15. (4 pts.) The function \( f(x) = \frac{x^4 - 5x^2 + 4}{x + 2} \) is not defined at a certain point. How should it be defined to make it continuous at that point?