TAKE-HOME EXAM 4

Solve the following problems showing all your work for full credit.

1. (5 pts.) Find parametric equations and a parameter interval for the motion of a particle that starts at \((a,0)\) and traces the ellipse \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\) twice counterclockwise.

2. (5 pts.) For the following parametric equation and parameter interval for the motion of a particle in the xy-plane identify the particle’s path by finding a Cartesian equation for it: \(x = 1 + \sin t, \ y = \cos t - 2, \ 0 \leq t \leq \pi\).

3. (5 pts.) Find an equation for the line tangent to the curve \(x = -\sqrt{t} + 1, \ y = \sqrt{3t}\) at the point \(t = 3\).

4. (5 pts.) Find the length of the curve \(x = t^3, \ y = \frac{3t^2}{2}, \ 0 \leq t \leq \sqrt{3}\).
5. (5 pts.) Find the area of the surface generated by revolving the curve $x = \frac{2}{3} t^{\frac{3}{2}}$, $y = 2\sqrt{t}$, $0 \leq t \leq \sqrt{3}$ about the y-axis.

6. (15 pts.) Replace the polar equations by equivalent Cartesian equations:
   a) $r^2 \sin 2\theta = 2$
   
b) $r = 3\cos\theta$
   
c) $r^2 = 4r \sin \theta$

7. (5 pts.) Sketch the graph of the polar equation $r = 3(1 - \cos \theta)$. 
8. (5 pts.) Find the points of intersection of the graphs of the equations $r = 1 + \cos \theta$ and $r = 3 \cos \theta$.

9. (5 pts.) Find the length of the curve $r = 2a \cos \theta$ over the indicated interval $\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

10. (5 pts.) Find the area inside one loop of the lemniscate $r^2 = 9 \sin 2\theta$. 
11. (25 pts.) Find the vertices, foci, directrixes, and sketch the graph of the curve

a) \( y^2 + 4y + 8x - 12 = 0 \)

b) \( 16x^2 + 25y^2 - 64x + 150y + 288 = 0 \)

c) \( 9x^2 - 4y^2 + 54x + 8y + 78 = 0 \)
12. (15 pts.) Find a polar equation for the conic with its focus at the pole.
   a) parabola, \( e = 1 \), directrix \( y = 1 \)

   b) ellipse, \( e = \frac{3}{4} \), \( y = -2 \)

   c) hyperbola with vertices \( \left( 1, \frac{3\pi}{2} \right) \) and \( \left( 9, \frac{3\pi}{2} \right) \)